Short Papers

Small-Signal Second-Harmonic Generation by a Nonlinear Transmission Line

KEITH S. CHAMPLIN, MEMBER, IEEE, AND DONALD R. SINGH, MEMBER, IEEE

Abstract —**Second-harmonic generation (SHG) by a relatively low-loss transmission line having a capacitive nonlinearity is treated with an extended small-signal analysis. This simple theory brings out the relevance of "phase matching" the fundamental- and second-harmonic waves and of reducing losses in order to optimize SHG. It is shown that maximum SHG will occur when the line is short compared with its "coherence length" and has radian electrical length equal to twice its "transmission Q" at the second-harmonic frequency. The product of a line's "transmission Q" and its "nonlinearity factor" should be maximized to obtain maximum efficiency and is, therefore, believed to be a useful figure of merit for comparing the SHG potential of different transmission-line implementations.**

I. Introduction

In recent years, nonlinear capacitances such as varactors and Schottky-barrier diodes have proven very useful as harmonic generators at microwave and submicrowave frequencies [1]. These devices have traditionally been lumped-parameter devices in which the nonlinear interactions take place over distances that are small in comparison with a wavelength. As frequency increases, however, lumped-parameter devices necessarily become smaller and ultimately reach fundamental limitations imposed by the increased series resistance and/or parasitic capacitance [2].

Nonlinear interactions in distributed devices take place over distances that may be comparable to, or even larger than, a wavelength [3], [4]. Accordingly, such devices are not subject to the same limitations as lumped-parameter devices. Instead, the fundamental problem of distributed devices is that of obtaining structures having sufficiently small losses that distributed nonlinearities can be properly exploited [5]. Recent work on Schottky-barrier and MIS transmission lines fabricated on semiconductor substrates has indicated that several such nonlinear structures may indeed demonstrate relatively low-loss propagation in their "slow wave" regimes of operation [6]–[15].

A simple small-signal treatment of second-harmonic generation (SHG) by waves propagating on a transmission line having a capacitance nonlinearity is presented below. The relevance of "phase matching" and the importance of minimizing losses are readily apparent from this simple theory. It is shown that for a line that is short compared with its "coherence length," maximum SHG will occur when the line's radian electrical length is equal to twice its "transmission Q" at the second-harmonic frequency. In addition, the conversion efficiency is found to be proportional to the square of the product of the line's transmission Q" and its "nonlinearity factor." Accordingly, this product is believed to be a useful figure of merit for comparing potential SHG of different transmission-line implementations.

Manuscript received April 24, 1985; revised October 18, 1985. This material is based in part upon work supported by the U.S. Army Research Office and the National Science Foundation under Grant No. ECS-83-16246.

The authors are with the Department of Electrical Engineering, University of Minnesota, Minneapolis, MN 55455.

IEEE Log NUmber 8406858.

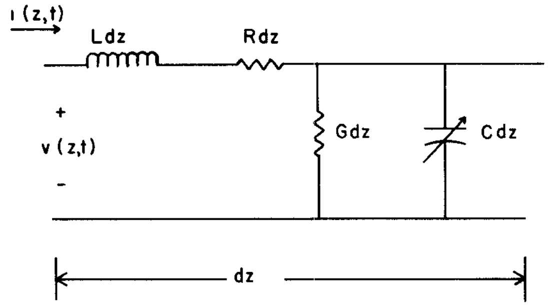

Fig. 1. Differential length of nonlinear transmission line.

II. Theory

Consider quasi-TEM waves, identified by voltage $v(z,t)$ and current $i(z,t)$, propagating along a two-conductor transmission line. Let L, R, C, and G represent the line's series inductance, series resistance, shunt capacitance, and shunt conductance, respectively, each per unit length. Fig. 1 depicts a differential length dz of the transmission line.

Assume that the shunt capacitance C of the line possesses a nonlinear voltage-charge relationship that can be expanded in a convergent Taylor's series

$$q(v) = q(V_0) + q'(V_0)\{v - V_0\} + \frac{1}{2!}q''(V_0)\{v - V_0\}^2 + \cdots \quad (1)$$

where q is the stored charge per unit length, and V_0 is the dc bias voltage. Kirchoff's laws written for a differential length of line yield

$$\frac{\partial i}{\partial z} = -Gv - \frac{\partial q}{\partial t} \quad (2)$$

and

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}. \quad (3)$$

Eliminating i between (2) and (3) leads to the second-order partial differential equation

$$\frac{\partial^2 v}{\partial z^2} - GRv - GL\frac{\partial v}{\partial t} - R\frac{\partial q}{\partial t} - L\frac{\partial^2 q}{\partial t^2} = 0. \quad (4)$$

If ω is the lowest frequency sinusoidal component of v, one can expand v and q in complex Fourier series as follows:

$$v(z,t) = V_0 + \sum_{n=1}^{\infty}\{V_n(z)e^{jn\omega t} + V_n^*(z)e^{-jn\omega t}\} \quad (5)$$

and

$$q(z,t) = Q_0(z) + \sum_{n=1}^{\infty}\{Q_n(z)e^{jn\omega t} + Q_n^*(z)e^{-jn\omega t}\}. \quad (6)$$

Substituting (5) and (6) into (4) and equating the terms yields

$$\frac{d^2}{dz^2} V_n - G\{R + jn\omega L\} V_n - (jn\omega)\{R + jn\omega L\} Q_n = 0 \quad \text{for } n = 1,2,3 \cdots. \tag{7}$$

Equation (7) represents a system of coupled linear differential equations relating the V_n and Q_n coefficients. By substituting (5) and (6) into (1) while assuming that harmonic voltage terms are small compared with V_1, one finds that, to the second order, these coefficients are also related by

$$\begin{aligned} Q_0 &= q(V_0) + q''(V_0) V_1 V_1^* \\ Q_1 &= q'(V_0) V_1 \\ Q_2 &= q'(V_0) V_2 + \frac{q''(V_0)}{2} V_1^2. \end{aligned} \tag{8}$$

Thus, the Q_N coefficients can be systematically eliminated between (7) and (8). For $n = 1$ and 2, this procedure leads to

$$\frac{d^2}{dz^2} V_1 - \gamma_1^2 V_1 = 0 \tag{9}$$

and

$$\frac{d^2}{dz^2} V_2 - \gamma_2^2 V_2 = \left[\frac{\gamma_2^2 (q''/2q')}{1 - j\tan\delta_2} \right] V_1^2 \tag{10}$$

where

$$\begin{aligned} \gamma_n &= \alpha_n + j\beta_n \\ &= \{G + jn\omega q'\}^{1/2} \{R + jn\omega L\}^{1/2} \end{aligned} \tag{11}$$

and

$$\delta_2 = \tan^{-1}\{G/2\omega q'\}. \tag{12}$$

Equation (9) is recognized as the usual homogeneous wave equation for the fundamental frequency voltage V_1. This fundamental voltage wave serves as the "forcing function" for the second-harmonic voltage wave V_2 according to (10). Assuming for simplicity that only the positive-traveling fundamental wave is excited, the solution to (9) is of the form

$$V_1(z) = V_1(0) e^{-\gamma_1 z}. \tag{13}$$

By substituting (13) into (10) and solving the resultant inhomogeneous differential equation, one obtains the second-harmonic particular integral

$$V_2(z) = \left[\frac{\gamma_2^2 K_N V_1^2(0) z e^{-\gamma_2 z}}{(2\gamma_1 + \gamma_2)(1 - j\tan\sigma_2)} \right] \left[\frac{e^{-(2\gamma_1 - \gamma_2)z} - 1}{(2\gamma_1 - \gamma_2) z} \right] \tag{14}$$

where K_N is a "nonlinearity factor" defined by

$$K_N \equiv (q''/2q'). \tag{15}$$

Equation (14) satisfies the boundary conditions $V_2(0) = V_2(\infty) = 0$.

If one utilizes (11) along with the assumption of relatively small losses

$$\begin{aligned} G &\ll 2\omega q' \\ R &\ll 2\omega L \end{aligned} \tag{16}$$

one can approximate (14) by

$$V_2(z) \cong (\tfrac{1}{2}) K_N V_1^2(0) (j\beta_2 z) e^{-\gamma_2 z} F(\phi) \tag{17}$$

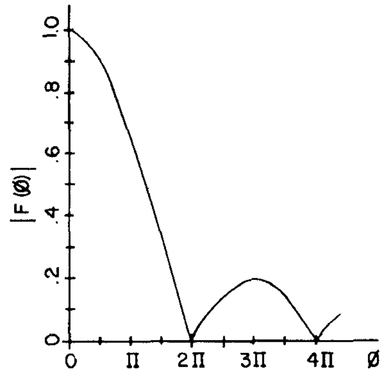

Fig. 2. Magnitude of "coherence function" $F(\phi)$

where $F(\phi)$ is a "coherence function" given by

$$F(\phi) = \left[\frac{e^{-j\phi} - 1}{j\phi} \right] = -e^{-j\phi/2} \left[\frac{\sin(\phi/2)}{\phi/2} \right] \tag{18}$$

with ϕ defined by

$$\phi \equiv -j(2\gamma_1 - \gamma_2) z. \tag{19}$$

If losses are small, ϕ is nearly real and can be written approximately

$$\phi \cong (2\beta_1 - \beta_2) z. \tag{20}$$

The magnitude of the "coherence function" $F(\phi)$ is plotted for real ϕ in Fig. 2. One sees that $|F|$ has a maximum of unity at $\phi = 0$ and passes through zero for $\phi = 2n\pi$. From this result, we define the "*coherence length*" z_c by

$$z_c \equiv 2\pi/(2\beta_1 - \beta_2). \tag{21}$$

For $z \ll z_c$, the fundamental frequency and second-harmonic waves propagate together and interfere constructively. As z approaches z_c, however, the two waves get "out of step" and destructive interference results. With pure TEM waves on lossless lines, $\beta_2 = 2\beta_1$ so that $z_c = \infty$. With quasi-TEM waves, however, z_c will be finite.

Let z_l be the total line length. For $z_l < (z_c/4)$, (17) leads to the approximation

$$|V_2(z_l)| \cong (\tfrac{1}{2}) K_N V_1^2(0) \beta_2 \{z_l e^{-\alpha_2 z_l}\} \tag{22}$$

which describes a propagating second-harmonic wave that is simultaneously growing linearly and attenuating exponentially. The optimum length of the transmission line is the length for which $\{z_l e^{-\alpha_2 z_l}\}$ is maximum. By differentiation, one finds that

$$z_{\text{opt}} = 1/\alpha_2. \tag{23}$$

Thus, the optimum electrical length of the line can be written

$$(\beta_2 z_{\text{opt}}) = 2Q_2 \tag{24}$$

where

$$Q_2 \equiv (\beta_2/2\alpha_2) \tag{25}$$

is the "transmission Q" of the line at the frequency ω_2. Substituting (24) into (22) yields the maximum second-harmonic output voltage

$$|V_2(z_{\text{opt}})| = (0.37) Q_2 K_N V_1^2(0). \tag{26}$$

The maximum conversion efficiency, obtained under these same conditions, is therefore

$$\mathrm{Eff}(z_{\mathrm{opt}}) = \frac{|V_2(z_{\mathrm{opt}})|^2}{V_1^2(0)} \times 100\%$$
$$= \{(0.37) Q_2 K_N V_1(0)\}^2 \times 100\% \qquad (27)$$

which is seen to be proportional to $(Q_2 K_N)^2$.

III. Conclusion

According to the simple small-signal theory presented above, the largest SHG of a relatively low-loss transmission line will occur when the transmission line satisfies both the *coherence* condition

$$z_l < (z_c/4) \qquad (28)$$

and the *optimum length* condition

$$(\beta_2 z_l) = 2Q_2. \qquad (29)$$

Further, the maximum obtainable SHG is seen to be proportional to the square of the $(Q_2 K_N)$ product. Accordingly, tradeoffs between the "transmission Q" and the "nonlinearity factor" may be possible which will maximize this product for a particular line implementation. The simple theory also shows that conversion efficiency is proportional to the square of the input voltage. Note, however, that (27) assumes $|V_2| \ll |V_1|$ and that third- and higher order terms in the Taylor's series expansion of (1) have been systematically ignored. Thus, this result can be rigorously justified only for small input signals satisfying

$$V_1(0) \ll \{3q''/q'''\}. \qquad (30)$$

References

[1] See, e.g., J. W. Archer, "Millimeter wavelength frequency multipliers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, p. 552, June 1981.

[2] See, e.g., K. S. Champlin and G. Eisenstein, "Cut-off frequency of submillimeter Schottky barrier diodes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 31–34, Jan. 1978

[3] V. P. Popov and T. A. Bickart, "*RC* transmission line with nonlinear controlled parameters—Small-signal characteristics," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 268–270, Mar. 1974

[4] V. P. Popov and T. A. Bickart, "*RC* transmission line with nonlinear resistance: Large-signal response computation," *IEEE Trans Circuits Syst.*, vol. CAS-21, pp. 666–671, Sept. 1974.

[5] K. Everszumrode, B. Brockmann, and D. Jager, "Efficiency of harmonic frequency generation along schottky contact microstrip lines," *Arch. Elek. Übertragung.*, vol. 31, pp. 212–215, 1977.

[6] D. Jager *et al.*, "Bias-dependent small-signal parameters of Schottky contact microstrip lines," *Solid-State Electron.*, vol. 17, pp. 777–783, 1974.

[7] H. Hasegawa *et al.*, "Properties of microstrip line on Si-SiO_2 systems," *IEEE Trans. Microwave Theory Tech.*, vol MTT-19, pp. 869–881, Nov. 1971.

[8] H. Hasegawa *et al.*, "Slow-wave propagation along a microstrip line on Si-SiO_2 systems," *Proc. IEEE*, vol. 59, pp. 297–299, Feb. 1971.

[9] G. W. Hughes, "Microwave properties of nonlinear MIS and Schottky barrier microstrip," *IEEE Trans. Electron Devices*, vol Ed-22, pp. 945–956, 1975.

[10] D. Jager, "Slow wave propagation along variable Schottky contact microstrip line," *IEEE Trans Microwave Theory Tech.*, vol MTT-24, pp 566–573, Sept. 1976.

[11] H. Hasegawa and H. Okizaki, "MIS and Schottky slow-wave coplanar striplines on GaAs substrates," *Electron. Lett*, vol. 13, pp. 663–664, 1977

[12] S. Seki and H Hasegawa, "Cross tie slow-wave coplanar waveguide on semiinsulating GaAs substrates," *Electron Lett*, vol. 17, pp 940–951, 1981.

[13] Y. Shih and T Itoh, "Analysis of printed transmission lines for monolithic integrated circuits," *Electron. Lett*, vol. 18, pp. 589–590, 1982.

[14] Y Fukuoka and T Itoh, "Analysis of slow-wave phenomena in coplanar waveguide on a semiconductor substrate," *Electron. Lett.*, vol. 18, pp. 589–590, 1982.

[15] R Sorrentino *et al.*, "Characteristics of metal-insulator-semiconductor coplanar waveguides for monolithic microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 410–416, Apr., 1984.

Transient Analysis of a Directional Coupler Using a Coupled Microstrip Slot line in Three-Dimensional Space

SHOICHI KOIKE, NORINOBU YOSHIDA, AND ICHIRO FUKAI

Abstract —**In recent MIC techniques, double-sided MIC has been studied because its advantages in propagation characteristics are greater than that of conventional MIC's. A coupled microstrip slotline is one of them. Its application to various circuit elements has often been discussed. But the coupled microstrip slotline is essentially three-dimensional structure, so the analysis demands a rigorous three-dimensional treatment. Also, the recent high-speed pulse technique demands analysis in the time domain. The present paper treats a directional coupler using the coupled microstrip slotline in three-dimensional space and time. The results of the directional coupler analysis is presented with the complicated time variation of the three-dimensional electromagnetic field. So, the mechanism of the directional coupling phenomena that is produced by the propagation characteristics of the even and odd modes is presented in the time domain. In particular, the instantaneous diagram of the Poynting vector details the energy flow in the transient process. For the analysis of the characteristics of the complex microwave device, these results present the utilities of the various field distributions that are obtained by the three-dimensional vector analysis in the time domain.**

I. Introduction

In recent MIC techniques, a double-sided MIC has been studied extensively because of its advantages in propagation characteristics over the conventional MIC. A coupled microstrip slotline is one of the fundamental structures of the double-sided MIC. The structure is as follows: the stripline is on one side of the dielectric substrate and the slotline is on the other side. The coupled stripline slotline has properties that the dispersion characteristics and the characteristic impedance are controlled sensitively and extensively by changing the geometrical dimension. The design of the directional coupler by use of the microstrip slotline was proposed by F. C. de Ronde in 1970, [1] its theoretical consideration was given by B. Schiek, [2], [3], the synthetic method of the design and experimental results were performed by H. Ogawa, [4], [5], and the design with compensation slotlines and the comparison with the experiment were presented by R. K. Hoffmann [6], [7]. The conventional MIC based on the stripline has been treated by a two-dimensional analysis. But, the coupled microstrip slotline is essentially a three-dimensional structure, so the analysis demands a rigorous three-dimensional treatment. Also, the recent development of high-speed pulse techniques demands the analysis in the time domain. The transient analysis of the electromagnetic field is not only useful in clarifying the field response but also yields information on the mechanism by

Manuscript received June 18, 1985; revised October 18, 1985.
The authors are with the Faculty of Engineering, Hokkaido University, Sapporo, 060 Japan.
IEEE Log Number 8406856.